

HOW TO SOLVE TRIGONOMETRIC EQUATIONS

Section - 8

8.1 While solving equations following points must be kept in mind.

- (i) Squaring should be avoided as far as possible. If squaring is done check for the extraneous roots.
- (ii) Never cancel equal terms containing 'unknown or variable' on two sides which are in product. It may cause root loss.
- (iii) The answer should not contain such values of θ which make any of the terms undefined.
- (iv) Domain should not change. If it changes, necessary corrections must be made.
- (v) Check that denominator is not zero at any stage while solving equations.

TYPE-I

Problems based on method of factorisation :

Working Rule :**Step I.** Take all the terms to L.H.S. i.e. make R.H.S. zero, then factorise L.H.S.**Step II.** Equate each factor to zero and solve.**Step III.** Check for root loss and extraneous roots.**Illustrating the Concepts :****(i)** Solve : $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ for general values of θ .**(ii)** Find the values of x satisfying $3 - 2 \cos x - 4 \sin x - \cos 2x + \sin 2x = 0$.

(i) Using $\tan(A+B)$, $\tan\theta + \tan 2\theta = \tan 3\theta (1 - \tan\theta \tan 2\theta)$ $\Rightarrow \theta = n\pi/3$,
Hence the equation can be written as : $\tan 3\theta (1 - \tan\theta \tan 2\theta) + \tan 3\theta = 0$ or $\tan\theta = \pm \frac{1}{\sqrt{2}}$ [where, $n \in \mathbb{I}$]
 $\tan 3\theta (2 - \tan\theta \tan 2\theta) = 0$ $\Rightarrow \theta = n\pi/3$,
 $\Rightarrow \tan 3\theta = 0$ or $\tan\theta \tan 2\theta = 2$ or $\theta = n\pi \pm \tan^{-1} \frac{1}{\sqrt{2}}$ [where, $n \in \mathbb{I}$]
 $\Rightarrow 3\theta = n\pi$ or $2 \tan^2\theta = 2(1 - \tan^2\theta)$

(ii) Put $\cos 2x = 2 \cos^2 x - 1$

$$\Rightarrow 3 - 2 \cos x - 4 \sin x - (2 \cos^2 x - 1) + \sin 2x = 0$$

$$\Rightarrow (4 - 4 \sin x) - 2 \cos^2 x - 2 \cos x + \sin 2x = 0$$

$$\Rightarrow 4(1 - \sin x) - 2(1 - \sin^2 x) - 2 \cos x (1 - \sin x) = 0$$

$$\Rightarrow (1 - \sin x)(2 - 2 \sin x - 2 \cos x) = 0$$

$$\text{Either } \sin x = 1 \quad \text{or} \quad \sin x + \cos x = 1$$

$$\Rightarrow \sin x = \sin \pi/2 \quad \text{or} \quad \sqrt{2} \cos(x - \pi/4) = 1$$

$$\Rightarrow x = n\pi + (-1)^n \pi/2 \quad \text{or} \quad x - \pi/4 = 2n\pi \pm \pi/4 \quad [\text{where, } n \in \mathbb{I}]$$

$$\Rightarrow x = n\pi + (-1)^n \pi/2 \quad \text{or} \quad x = 2n\pi \pm \pi/4 + \pi/4 \quad [\text{where, } n \in \mathbb{I}]$$

$$\Rightarrow x = n\pi + (-1)^n \pi/2 \quad \text{or} \quad x = 2n\pi, 2n\pi + \pi/2 \quad [\text{where, } n \in \mathbb{I}]$$

Combining the two, we get :

$$x = 2n\pi, 2n\pi + \pi/2.$$

Illustration - 37 The number of solutions of $\cos x + \cos 2x + \cos 4x = 0$, where $0 \leq x \leq \pi$.

- (A) 2 (B) 3 (C) 4 (D) None of these

SOLUTION : (C)

$$\cos x + (\cos 2x + \cos 4x) = 0$$

$$\Rightarrow \cos x + 2 \cos 3x \cos x = 0$$

$$\Rightarrow \cos x (1 + 2 \cos 3x) = 0$$

$$\Rightarrow \cos x = 0$$

$$\text{or } 1 + 2 \cos 3x = 0$$

$$\Rightarrow \cos x = 0$$

$$\text{or } \cos 3x = -1/2 = \cos 2\pi/3$$

$$\Rightarrow x = (2n + 1)\pi/2$$

$$\text{or } 3x = 2n\pi \pm 2\pi/3 \quad [\text{where, } n \in \mathbb{I}]$$

$$\Rightarrow x = (2n + 1)\pi/2,$$

$$\text{or } x = 2n\pi/3 \pm 2\pi/9, \quad [\text{where, } n \in \mathbb{I}]$$

This is the general solution of the equation. To get particular solution satisfying $0 \leq x \leq \pi$, we will substitute integral values of n .

$$(i) \quad n = 0 \quad \Rightarrow \quad x = \pi/2, \pm 2\pi/9$$

$$(ii) \quad n = 1 \quad \Rightarrow \quad x = 3\pi/2, 8\pi/9, 4\pi/9$$

$$(iii) \quad n = 2 \quad \Rightarrow \quad x = 5\pi/2, 14\pi/9, 10\pi/9$$

(greater than π)

$$(iv) \quad n = -1 \quad \Rightarrow \quad x = -\pi/2, -2\pi/3 \pm 2\pi/9$$

(less than 0)

Hence the values for $0 \leq x \leq \pi$ are $x = \pi/2, 2\pi/9, 4\pi/9, 8\pi/9$.

Illustration - 38 Number of solutions of the equation $\sin x = \cos 4x$ for $0 \leq x \leq \pi$.

- (A) 3 (B) 2 (C) 4 (D) None of these

SOLUTION : (A)

$$\sin x = \cos 4x$$

$$\Rightarrow \cos 4x = \cos (\pi/2 - x)$$

$$\Rightarrow 4x = 2n\pi \pm (\pi/2 - x)$$

$$\text{Either } 4x = 2n\pi + \pi/2 - x$$

$$\text{or } 4x = 2n\pi - \pi/2 + x$$

$$\Rightarrow x = 2n\pi/5 + \pi/10$$

$$\text{or } x = 2n\pi/3 - \pi/6$$

This is the general solution of the equation. To get particular solution satisfying $0 \leq x \leq \pi$, we will substitute integral values of n .

$$(i) \quad n = 0 \quad \Rightarrow \quad x = \pi/10, -\pi/6$$

$$(ii) \quad n = 1 \quad \Rightarrow \quad x = \pi/2$$

$$(iii) \quad n = 2 \quad \Rightarrow \quad x = 9\pi/10, 7\pi/6$$

$$(iv) \quad n = 3 \quad \Rightarrow \quad x = 13\pi/10, 11\pi/6$$

(greater than π)

$$(v) \quad n = -1 \quad \Rightarrow \quad x = -3\pi/10, -5\pi/6$$

(less than 0)

Hence the required solution for $0 \leq x \leq \pi$ is $x = \pi/10, \pi/2, 9\pi/10$.

Illustration - 39 Number of solutions of the equation $\sec 4x - \sec 2x = 2$; in $-\pi \leq x \leq \pi$.

- (A) 8 (B) 10 (C) 12 (D) 14

SOLUTION : (B)

$$\sec 4x - \sec 2x = 2$$

$$\Rightarrow \frac{1}{\cos 4x} - \frac{1}{\cos 2x} = 2$$

$$\Rightarrow \cos 2x - \cos 4x = 2 \cos 2x \cos 4x$$

$$\Rightarrow \cos 2x - \cos 4x = \cos 6x + \cos 2x$$

$$\Rightarrow \cos 6x + \cos 4x = 0$$

$$\Rightarrow 2 \cos 5x \cos x = 0$$

$$\text{Either } \cos 5x = 0 \quad \text{or} \quad \cos x = 0$$

$$\Rightarrow 5x = 2n\pi \pm \pi/2$$

$$\text{or } x = 2n\pi \pm \pi/2 \quad [\text{where, } n \in \mathbb{I}]$$

$$\Rightarrow x = 2n\pi/5 \pm \pi/10$$

$$\text{or } x = 2n\pi \pm \pi/2 \quad [\text{where, } n \in \mathbb{I}]$$

This is the general solution of the equation. To get particular solution satisfying $-\pi \leq x \leq \pi$, we will substitute integral values of n .

Consider $x = 2n\pi/5 \pm \pi/10$:

$$n = 0 \quad \Rightarrow \quad x = \pm \pi/10$$

$$n = \pm 1 \quad \Rightarrow \quad x = \pm \pi/2, \pm 3\pi/10$$

$$n = \pm 2 \quad \Rightarrow \quad x = \pm 9\pi/10, \pm 7\pi/10$$

Consider $x = 2n\pi \pm \pi/2$:

$$n = 0 \quad \Rightarrow \quad x = \pm \pi/2$$

These are the only values of x in $[-\pi, \pi]$.

Illustration - 40 The solutions of the given equation $\cos \theta \cos 2\theta \cos 3\theta = 1/4$ are $\theta = (2n+1)A$ and $n\pi + B$, then 'A and B' are :

(A) $\pi, \frac{\pi}{3}$

(B) $\frac{\pi}{8}, \frac{\pi}{3}$

(C) $\frac{\pi}{2}, 0$

(D) None of these

SOLUTION : (B)

$$4 \cos \theta \cos 2\theta \cos 3\theta = 1$$

$$\text{or, } (2 \cos 3\theta \cos \theta) (2 \cos 2\theta) = 1$$

$$\text{or, } (\cos 4\theta + \cos 2\theta) (2 \cos 2\theta) - 1 = 0$$

$$\text{or, } 2 \cos 4\theta \cos 2\theta + 2 \cos^2 2\theta - 1 = 0$$

$$\text{or, } 2 \cos 4\theta \cos 2\theta + \cos 4\theta = 0$$

$$\text{or, } \cos 4\theta [2 \cos 2\theta + 1] = 0$$

$$\text{Either } \cos 4\theta = 0, \quad \text{or} \quad 2 \cos 2\theta + 1 = 0.$$

$$4\theta = (2n+1) \frac{\pi}{2}$$

$$\text{or } \cos 2\theta = -\frac{1}{2} = \cos \frac{2\pi}{3}$$

$$\Rightarrow \theta = (2n+1) \frac{\pi}{8}$$

$$\text{or } 2\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\theta = n\pi \pm \frac{\pi}{3} \quad [\text{where, } n \in \mathbb{I}]$$

Hence,

$$\theta = (2n+1) \frac{\pi}{8}, n\pi \pm \frac{\pi}{3} \text{ where } n = 0, \pm 1, \pm 2, \dots$$

TYPE-II

Equations of the form $a \cos \theta + b \sin \theta = c$

Working Rule :

(i) Divide by $\sqrt{a^2 + b^2}$ on both sides and get $\frac{a}{\sqrt{a^2 + b^2}} \cos \theta + \frac{b}{\sqrt{a^2 + b^2}} \sin \theta = \frac{c}{\sqrt{a^2 + b^2}}$.

(ii) Write L.H.S. as $\sqrt{a^2 + b^2} \cos(\theta - \alpha)$ [where $\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$ and $\sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$]

(iii) Simplify the equation.

Illustration - 41 The number of solutions of equation $\sqrt{3} \sin x + \cos x = 1$ in the interval $0 \leq x \leq 2\pi$ are :

- (A) 3 (B) 2 (C) 4 (D) None of these

SOLUTION : (A)

$$\sqrt{3} \sin x + \cos x = 1$$

$$\Rightarrow 2 \left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x \right) = 1$$

[Multiplying and divide by $\sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2}$]

$$\Rightarrow 2 (\cos \pi/3 \cos x + \sin \pi/3 \sin x) = 1$$

$$\Rightarrow 2 \cos (x - \pi/3) = 1$$

$$\Rightarrow \cos (x - \pi/3) = \cos \pi/3$$

$$\Rightarrow x - \pi/3 = 2n\pi \pm \pi/3$$

$$\Rightarrow x = 2n\pi + 2\pi/3, x = 2n\pi \quad [\text{where, } n \in \mathbb{I}]$$

This is the general solution of the equation. To get particular solution satisfying $0 \leq x \leq 2\pi$ we will substitute integral values of n .

$$(i) \quad n = 0 \Rightarrow x = 0 \quad \text{or} \quad 2\pi/3$$

$$(ii) \quad n = 1 \Rightarrow x = 2\pi + 2\pi/3 \quad \text{or} \quad 2\pi$$

$$(iii) \quad n = 2 \Rightarrow x = 4\pi + 2\pi/3$$

or 4π (greater than π)

$$(iv) \quad n = -1 \Rightarrow x = -4\pi/3$$

or -2π (less than 0)

Hence the required values of x are 0, $2\pi/3$, 2π .

Illustration - 42 Solve the equation : $\sin x + \cos x = \sin 2x - 1$.

SOLUTION :

$$\text{Let } t = \sin x + \cos x$$

$$\Rightarrow t^2 = 1 + 2 \sin x \cos x$$

$$\Rightarrow \sin 2x = t^2 - 1$$

$$\text{Hence the given equation is } t = (t^2 - 1) - 1$$

$$\Rightarrow t^2 - t - 2 = 0$$

Solving the equation to get :

$$(t - 2)(t + 1) = 0$$

$$\Rightarrow t = 2 \quad \text{or} \quad t = -1$$

$$\Rightarrow \sin x + \cos x = 2$$

$$\text{or} \quad \sin x + \cos x = -1$$

$$\Rightarrow \sqrt{2} \cos (x - \pi/4) = 2$$

$$\text{or} \quad \sqrt{2} \cos (x - \pi/4) = -1$$

$$\Rightarrow \cos (x - \pi/4) = \sqrt{2}$$

$$\text{or} \quad \cos (x - \pi/4) = -\frac{1}{\sqrt{2}}$$

As $-1 \leq \cos \theta \leq 1$, $\cos (x - \pi/4) = \sqrt{2}$ is impossible.

$$\Rightarrow \cos (x - \pi/4) = -\frac{1}{\sqrt{2}} \text{ is the only possibility.}$$

$$\Rightarrow \cos (x - \pi/4) = \cos (\pi - \pi/4)$$

$$\Rightarrow x - \pi/4 = 2n\pi \pm 3\pi/4 \quad [\text{where, } n \in \mathbb{I}]$$

$$\Rightarrow x = 2n\pi \pm 3\pi/4 + \pi/4 \text{ is the general solution.}$$

TYPE-III

Trigonometric Inequality

Working Rule :

While solving inequations involving trigonometric functions, it is best to use graphs of trigonometric functions.

Illustrating the Concepts :

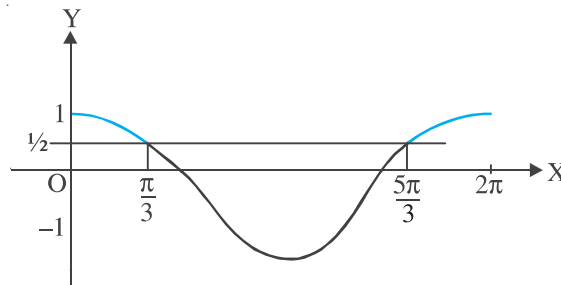
Solve $\cos x \geq \frac{1}{2}$.

We construct the graph $y_1 = \cos x$, $y_2 = +\frac{1}{2}$.

Now, on this graph we want those values of x for which graph of $y = \cos x$ is more than the graph of $y = \frac{1}{2}$ which is shown by color part of the graph.

We get solution as $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$ or general solution is

$$2n\pi \leq x \leq \frac{\pi}{3} + 2n\pi \text{ and } 2n\pi + \frac{5\pi}{3} \leq x \leq 2\pi + 2n\pi.$$

**Illustration - 43**

The solution of the inequality $\sin x + \cos 2x > 1$ if $0 \leq x \leq \frac{\pi}{2}$

- (A) $0 < x < \frac{\pi}{4}$ (B) $\frac{\pi}{3} < x < \frac{\pi}{2}$ (C) $0 < x < \frac{\pi}{6}$ (D) $\frac{\pi}{4} < x < \frac{\pi}{2}$

SOLUTION : (C)

$$\text{Let } \sin x = t \Rightarrow \cos 2x = 1 - 2t^2$$

$$\Rightarrow \text{The inequality is : } t + 1 - 2t^2 > 1$$

$$\Rightarrow 2t^2 - t < 0$$

$$\Rightarrow t(2t - 1) < 0$$

$$\Rightarrow (t - 0)(t - 1/2) < 0$$

$$\Rightarrow 0 < t < 1/2$$

$$\Rightarrow 0 < \sin x < 1/2$$

In $0 \leq x \leq \pi/2$, this means that $0 < x < \pi/6$ is the solution.

TYPE-IV

Simultaneous Equations

- (i) Two equations in one variable.

Working Rule :

- (i) If two equations in one unknown (say x) are given, then solve given equations separately for x lying between 0 and 2π .
- (ii) Select the value of x for which both the equations are satisfied.
- (iii) Add $2n\pi$ to get general solution.

Illustrating the Concepts :

Solve $\cos x = \frac{1}{\sqrt{2}}$, and $\tan x = -1$.

$$\cos x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}, \frac{7\pi}{4} \text{ in } (0, 2\pi)$$

$$\tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4} \text{ in } (0, 2\pi)$$

$$\therefore \text{Common value of } x = \frac{7\pi}{4}$$

Now for general solution add a factor of $2n\pi$.

$$\therefore \text{General solution is } x = 2n\pi + \frac{7\pi}{4}.$$

TYPE-IV**(ii)** Two equations in two variable.**Working Rule :**

Step I. Find $A + B$ and $A - B$ between 0 and 2π from the two given equations. Impose the condition that A and B must be smallest and positive, since A and B are positive angles, $A + B > A - B$. Find $A + B$ and $A - B$ accordingly.

Step II. Solve the two equation to get A and B .

Step III. Then generalised it with the help of common period. i.e. adding a factor of $n\pi$.

Illustrating the Concepts :

If $\tan(A - B) = 1$, $\sec(A + B) = 2/\sqrt{3}$, calculate the smallest positive values and the most general values of A and B .

Smallest Positive Values

Let $A, B \in (0, 2\pi)$

$$\Rightarrow (A + B) > (A - B)$$

$$\text{Now } \tan(A - B) = 1 \quad \Rightarrow \quad (A - B) = \pi/4, 5\pi/4$$

$$\sec(A + B) = 2/\sqrt{3} \quad \Rightarrow \quad (A + B) = \pi/6, 11\pi/6$$

As $(A + B) > (A - B)$, there are two possibilities :

$$\text{(i)} \quad A - B = \pi/4 \quad \text{and} \quad A + B = 11\pi/6$$

$$\text{(ii)} \quad A - B = 5\pi/4 \quad \text{and} \quad A + B = 11\pi/6$$

From (i), we get :

$$A = \frac{25\pi}{24} \quad \text{and} \quad B = \frac{19\pi}{24}$$

$$\text{From (ii), we get : } A = \frac{37\pi}{24} \quad \text{and} \quad B = \frac{7\pi}{24}$$

General Values

$$\tan(A - B) = 1 \quad \Rightarrow \quad A - B = n\pi + \pi/4$$

$$\sec(A + B) = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad A + B = 2k\pi \pm \frac{\pi}{6}$$

$$\text{Taking } A - B = n\pi + \frac{\pi}{4} \quad \text{and} \quad A + B = 2k\pi + \frac{\pi}{6} \quad \text{we get :}$$

$$A = \frac{(2k + n)\pi}{2} + \frac{5\pi}{24} \quad \text{and} \quad B = \frac{(2k - n)\pi}{2} - \frac{\pi}{24}$$

$$\text{Taking } A - B = n\pi + \frac{\pi}{4} \quad \text{and} \quad A + B = 2k\pi - \frac{\pi}{6} \quad \text{we get :}$$

$$A = \frac{(2k + n)\pi}{2} + \frac{\pi}{24} \quad \text{and} \quad B = \frac{(2k - n)\pi}{2} + \frac{5\pi}{24}$$

TYPE-V

Miscellaneous Type

- (i) Whenever the terms on the two sides (L.H.S. and R.H.S.) of the equation are of different nature, We use inequality method for testing whether the equation has any real solution or not.

Working Rule :

- (i) Let $y =$ each side of the equation and break the equation in two parts.
 (ii) Form the inequality for y taking L.H.S. of the equation and also for R.H.S. of the equation. If there is any value of y satisfying both the inequalities, then the equation will have real solution and if there is no such y , the equation will have no real solution.

Illustrating the Concepts :

Show that the equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + x^{-2}$ for $0 < x \leq \frac{\pi}{2}$ has no real solution.

Let $y_1 = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x \quad \dots(i)$

and $y_2 = x^2 + x^{-2} \quad \dots(ii)$

From (i), $y_1 = 2\cos^2\frac{x}{2} \cdot \sin^2 x = (1 + \cos x) \cdot \sin^2 x = (< 2) \times (\leq 1)$

$< 2 \quad \left[\text{For } 0 < x \leq \frac{\pi}{2}, 0 \leq \cos x < 1 \text{ and } 0 < \sin x \leq 1 \right]$

i.e. $y < 2 \quad \dots(iii)$

From (ii), $y_2 = x^2 + x^{-2} = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \geq 2 \quad [\because x > 0]$

i.e. $y_2 \geq 2 \quad \dots(iv)$

No value of y can be obtained satisfying (iii) and (iv) simultaneously,

$\Rightarrow$ no real solution of the equation exists.

- (ii) Whenever the equation contains power terms.

Working Rule :

- (i) ★ Equate the base if possible.
 (ii) If not possible to equate the base then take log of both side

Illustration - 44 Find the values of x in $(-\pi, \pi)$ which satisfy the equation

$$8^1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \dots \text{to infinity} = 4^3.$$

- (A) 2 (B) 4 (C) 4 (D) 8

SOLUTION : (B)

$$8^1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \dots \text{to infinity} = 8^2$$

$$\Rightarrow 1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \dots \text{to infinity} = 2$$

This is an infinite geometric series with first term 1 and common ratio $|\cos x|$.

$$\Rightarrow \frac{1}{1-|\cos x|} = 2 \quad [\text{by using sum of infinite geometric series with first term } a \text{ and common ratio } r \text{ is } \frac{a}{1-r} \text{ if } |r| < 1]$$

$$\text{or, } 1-|\cos x| = \frac{1}{2} \Rightarrow |\cos x| = \frac{1}{2} \Rightarrow \cos x = \pm \frac{1}{2}$$

$$\text{When } \cos x = \frac{1}{2} = \cos \frac{\pi}{3}, x = 2n\pi \pm \frac{\pi}{3}$$

$$\text{When } \cos x = -\frac{1}{2} = \cos \frac{2\pi}{3}, x = 2n\pi \pm \frac{2\pi}{3}$$

The value of x in the given interval $(-\pi, \pi) = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}$.

TYPE-VI

Whenever terms are in sin, cos in power 1, all terms connected with plus sign and number of terms in L.H.S. is equal to the number in R.H.S. [with (+) or (-) sign] then each term must have its extremum value.

Working Rule :

In such problems each term will be (+1) when the number in R.H.S. is (+)ve and each term will be (-1) when the number in R.H.S. is (-)ve.

Illustrating the Concepts :

Solve the equation $\sin 6x + \cos 4x + 2 = 0$.

$$\sin 6x + \cos 4x + 2 = 0 \text{ or } \sin 6x + \cos 4x = -2 \quad \dots(i)$$

$$\Rightarrow \sin 6x = -1 \text{ and } \cos 4x = -1 \text{ both satisfied simultaneously}$$

$$\text{Now, } \sin 6x = -1 = \sin \frac{3\pi}{2} \Rightarrow 6x = 2n\pi + \frac{3\pi}{2} \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{4}, \text{ where } n \in I.$$

$$\Rightarrow \text{Values of } x \text{ between } 0 \text{ and } 2\pi \text{ are } \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$\text{Also, } \cos 4x = -1 = \cos \pi \Rightarrow 4x = 2n\pi + \pi$$

$$\therefore x = \frac{n\pi}{2} + \frac{\pi}{4}, \text{ where } n \in I.$$

$$\Rightarrow \text{Values of } x \text{ between } 0 \text{ and } 2\pi \text{ are } \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

Hence, values of x lying between 0 and 2π satisfying both the equations are $\frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore$ General solution will be given by

$$x = 2n\pi + \frac{\pi}{4} \quad \text{or,} \quad 2n\pi + \frac{5\pi}{4} \quad [\text{where } n \in I]$$

$$\text{i.e. } x = 2n\pi + \frac{\pi}{4} \quad \text{or,} \quad (2n+1)\pi + \frac{\pi}{4}$$

Combining these two results, to get :

$$x = m\pi + \frac{\pi}{4}, \quad [\text{where } n \in I]$$